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HP 12C Time Value of Money (TVM) Basics

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Interest and How It Works

The whole point of having a financial calculator is to do financial calculations, of course. And the whole world of finance is based upon one simple fact: *Borrowed money earns interest over time*. So it's best to begin with a quick reminder about interest and how it works (and there are some subtleties here that you may not have considered before, so follow along).

Basically, there are two kinds of interest: ***Simple*** interest and ***compound*** interest. Look at each kind.

Simple interest is far less common nowadays. With simple interest, the amount of money charged per period—as interest—is defined as a set percentage of the *amount originally loaned*. So, if $100.00 is loaned for 6 months at 1% per month, then the amount owed for interest will be exactly $6.00. For each month of the loan, the borrower must pay 1% of the original $100.00, so that's $1.00 per month, for 6 months. Simple to understand, simple to calculate: simple interest.

Compound interest is far more widely used. With compound interest, the amount of money charged per period—as interest—is defined as a set percentage of the *amount owed at the beginning of that period*. (Notice how that differs from simple interest: Simple interest is a percentage of the amount originally loaned—originally owed—but compound interest is a percentage of the amount owed at some other point in time, and that point changes.) So look at that same $100.00 loan for 6 months at 1% per month, but now assume that it's compound interest:

The interest for the Month 1 would be 1% of $100.00, or $1.00.
The amount owed at the end of Month 1 would therefore be $101.00.
The interest for the Month 2 would be 1% of $101.00, or $1.01.
The amount owed at the end of Month 1 would therefore be $102.01.
The interest for the Month 3 would be 1% of $102.01, or $1.02.
The amount owed at the end of Month 3 would therefore be $103.03. And so on….

Thus interest is earned on other interest earned previously—hence the name, compound interest. This is the form of interest used by the HP 12C's financial (TVM) registers, and it's the main focus here. Whenever you see the word "interest" here, you may assume that it's compound interest.

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That may all seem pretty elementary, but here are those subtle points that are often overlooked:

- First of all, there must be a specific time period defined and a specific interest rate (usually expressed as a percentage of the current balance) defined for that time period. The example above specifically stated that interest accrued at "1% per month." So the interest period was 1 month; the interest rate was 1%.

  But often you'll see interest rates quoted on an annual basis, even though they actually are defined on a monthly basis. A bank or other lender, for example, would quote an interest rate of 12% per year for the above loan, but that is only a convenient approximation—called a nominal rate. The bank would still use 1% per month in its actual calculations.

  So don't confuse a nominal annualized rate, quoted for the sake of convenience, for the actual rate. Always make sure you know the defined time period and defined interest rate for that period. That's what will really drive the math—and that's what the HP 12C needs to work with.

- Secondly, notice that in the above loan example, nobody said anything about how much was owed after, say, 2.5 months, or 3.75 months, or 4.19 months, etc. The interest definitions—the time period and percentage—specify only what the loan balance (amount owed) will be at one point in the period (the end). If you want to know the loan balance at any other time during the period, you'll need other definitions.

With these ideas in mind, look now at financial calculations on the HP 12C.

The Time Value of Money (TVM) calculations performed by the HP 12C's financial registers all involve an investment: Money lent; money returned (with some interest earned, hopefully). The sums lent and returned can vary a lot, both in amounts and timing. So, as with most complex situations, the best way to understand any given investment—to simplify it as much as possible—is to *draw a picture*.

First, you draw the picture for yourself. Then you draw it for your calculator….

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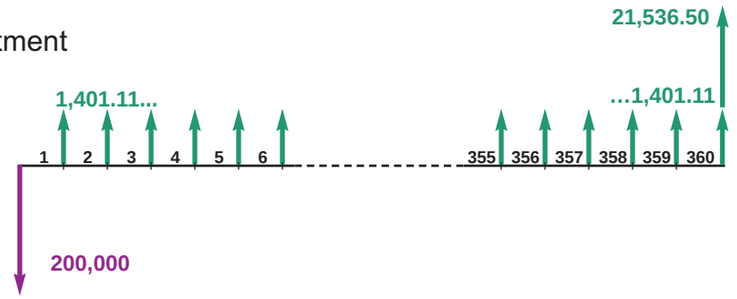
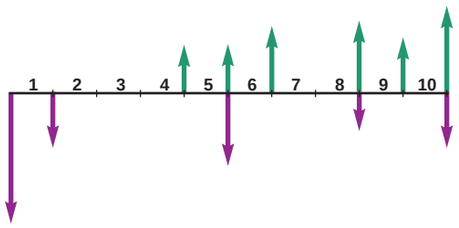
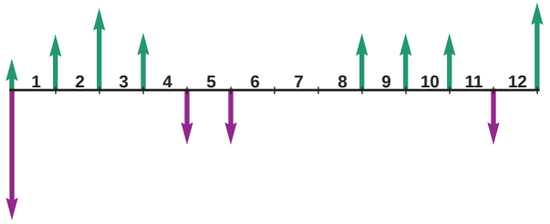
Drawing the Picture For Yourself: Cash Flow Diagrams

A cash flow diagram is a picture of an investment situation. Here's a typical example—a loan:



This is not a complicated situation, really. (A lender advances an amount of money—all in one lump sum—to a borrower, in exchange for steady, periodic payments for an agreed upon number of periods, with a lump sum payoff at the end) But other situations can get more involved, such as these:





But no matter how complicated an investment situation appears, you can reduce it to its bare essentials on a cash flow diagram, if you follow five rules when drawing the diagram.

Rule 1: "Either a borrower or a lender be!" Pick one perspective and stick with it while drawing the diagram. You'll draw the same investment situation entirely differently (upside down, in fact) depending on whether you're the lender or borrower. For example, if you're buying money market shares or a CD, you're a lender. If you're taking out a mortgage on your house, you're a borrower.

Rule 2: Once you've picked your perspective, the directions of the vertical arrows then denote the directions of the transactions. An **upward** arrow means that you **receive** money; a **downward** arrow means that you **pay** money. And note how the above diagrams show some of each. *You don't have an investment unless there's at least one cash flow in each direction—at least one **investment** and at least one **return**.*

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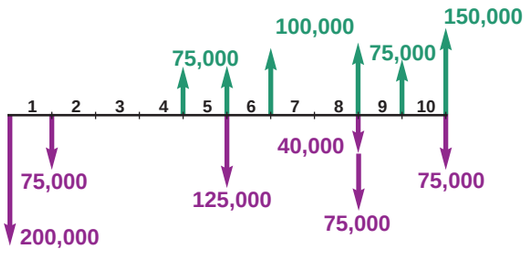
Rule 3: For the sake of clarity, let the lengths of the vertical arrows reflect the relative amounts of money flowing. A larger cash flow should be shown with a longer arrow. This will help to eliminate mistakes and omissions on the diagram. Again, look at the above examples.

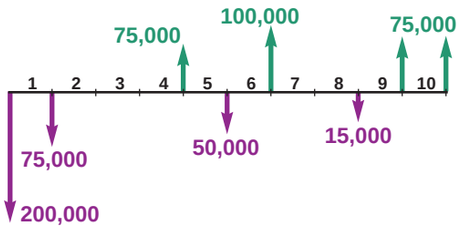
Rule 4: The horizontal line on the diagram is a time line. It represents time flowing (left to right). And this line should be marked at regular intervals to represent the defined interest period—so you can see exactly how often interest is compounding. Again, look at the above examples.

Rule 5: Whenever there is more than one cash flow occurring at the same point in time, you can add all those cash flows together into one net cash flow. In other words,

this…



…is equivalent to this.



If you follow those rules for drawing cash flow diagrams for yourself, you will be able, quickly and easily, to reduce a seemingly complex investment situation to its bare essentials—the whole point of a diagram.

But the beauty of the cash flow diagram doesn't end there. It has one other property that makes it even more useful:

You can ***adjust*** a cash flow diagram—move cash flows around—in ways that maintain the equivalence of the situation but which make the picture even simpler.

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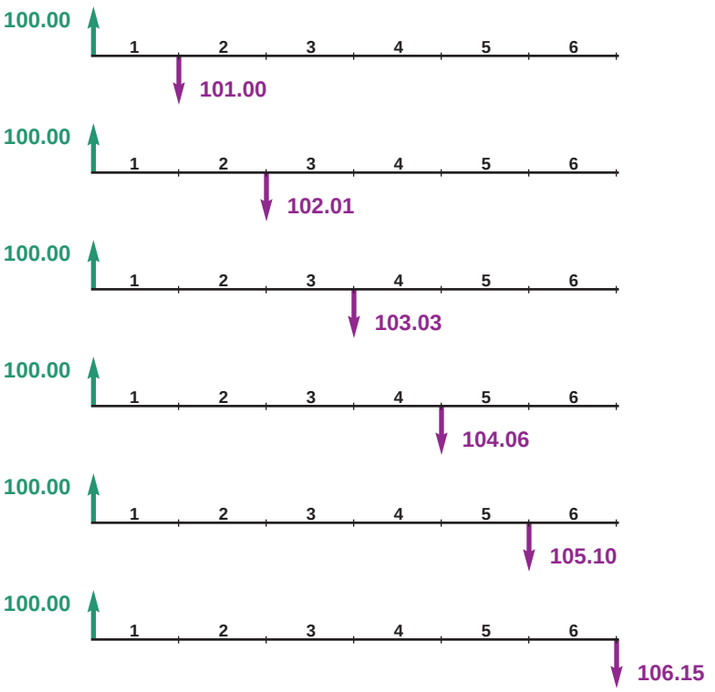
Look again at that $100.00 loan for 6 months. At the end of each month, the total amount owed had increased. (But that only makes sense, right? More time passing means more interest accruing.)

As of the end of:	The amount owed is:
Month 1	$101.00
Month 2	$102.01
Month 3	$103.03
Month 4	$104.06
Month 5	$105.10
Month 6	$106.15

The point is, if you could repay the loan at the end of any month, you could have six different pictures of the same loan, *but each picture would be accurate.* That is, in each case, the lender could consider it a fair transaction that had honored his/her loan terms completely: "$100.00 lent at 1% per month interest."



So it's like playing "What If?" with the situation: "What if I paid after the first month? What if I waited 4 more months?" Etc. And as you ponder, you're sliding that repayment to and fro across the time line. But no matter where you put it, it's *accurate as long as you adjust the repayment amount to account for the interest accrued to that point in time.*

In an uncomplicated loan like this, it's easy to see how the interest rate determines how that repayment amount shrinks or grows as you slide it around on the time line. But this is true for any cash flow on any diagram. *You can move any flow forward or backward in time and maintain complete accuracy in the picture, provided that you let that flow amount shrink or grow according to the interest rate in effect. That's the* ***Time Value of Money****: If the time changes, so does the money.*

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Drawing the Picture for Your HP 12C: The Financial (TVM) Registers

Now that you have a clear and consistent method for visually representing an investment situation to yourself, how do you draw that picture for your calculator? That is, how do you translate a cash flow diagram into numbers that you can enter into your HP 12C? The machine's financial registers—also called the TVM (Time Value of Money) registers—are designed to help you do just that. (There's more basic information available about those and all other HP 12C registers, too.)

The five financial registers correspond to the parts of a cash flow diagram (see the diagram below):

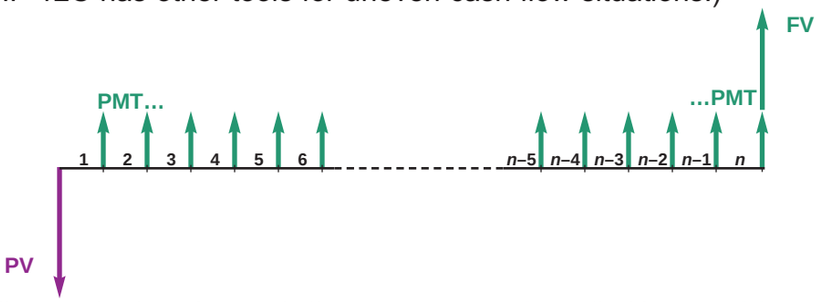
n = the **n**umber of defined interest periods across the entire timeline.

i = the defined **i**nterest rate (as a percentage) earned each period.

PV = the **P**resent **V**alue. This is the net cash flow that occurs at the left end of the time line *over and above any PMT that may occur there* (see below).

FV = the **F**uture **V**alue.This is the net cash flow that occurs at the right end of the time line *over and above any PMT that may occur there* (see below).

PMT = the **P**ay**M**en**T**. This is the cash flow that occurs once each period. The PMT amount can be zero, but it *must be the same for every period* (i.e. like a typical installment loan). If the cash flow situation doesn't have level cash flows, exactly one per period (and at the same point in the period), the financial registers won't give correct results. (The HP 12C has other tools for uneven cash flow situations.)



And how do you indicate whether the cash flows are **up** or **down**? *Use **positive** numbers for "**up**" cash flows; **negative** numbers for "**down**" cash flows.* (The CHS key changes the sign of a number.)

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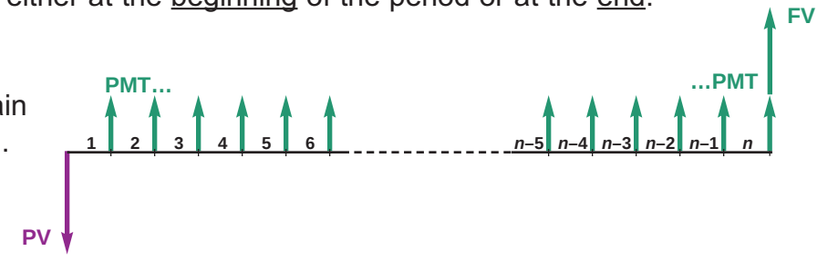
HP 12C Time Value of Money (TVM) Basics

BEGIN vs. END Mode

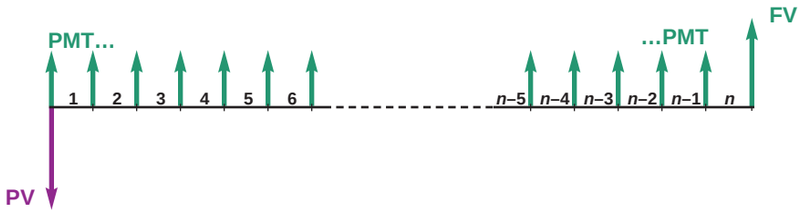
That's all there is to using the financial registers to draw a cash flow diagram for your HP 12C—except one very important point. The PMT amount—that steady, once-per-period cash flow amount—has a question attached to it: *When during each interest period does the PMT occur?*

The HP 12C gives you two choices: either at the beginning of the period or at the end.

The above diagram (shown here again has a PMT at the end of each period.



But here's how the situation would look if the PMT were to occur at the beginning of each period.



Does it make a difference in your calculations? Absolutely. Notice that in the second case, the loan repayment starts sooner (immediately when the money is lent, in fact), so there's going to be a little less interest earned. *That will change all the numbers.*

To indicate which assumption (PMT at the beginning or end of the period) you want the HP 12C to use, you set a mode on the calculator: For any situation where the PMT is at the beginning of the period (a less common situation), you set BEGIN by pressing g BEG (so that the little **BEGIN** annunciator appears in the display). If the PMT happens at the end of the period (more conventional), be sure that **BEGIN** is *not* showing, by pressing g END, if necessary. (There is no END annunciator; if you don't see the **BEGIN** annunciator the machine is in END mode.)

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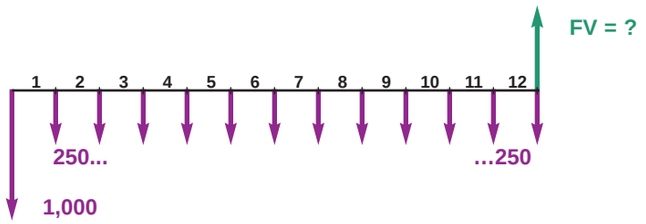
HP 12C Time Value of Money (TVM) Basics

Simple First Examples of Playing "What If"

It's time for some examples of the use of the financial registers. The whole idea is that you know four out of the five values (n, I, PV, PMT and FV), so you store those in the HP 12C's financial registers, then ask the calculator to solve for the remaining value. *Any of the five quantities can be the unknown value*—so it makes it very easy and quick to play "What If" and explore many hypothetical investment situations.

Problem: On January 1, you invest $1000 to open an account earning 6% interest (compounded monthly), then $250 more at the end of each month. Find the year-end balance.

Solution: Here's the situation on a cash flow diagram, from your viewpoint as the investor (lender).



Now solve it on the HP 12C:

Keystrokes	Comments
g END	The PMT amount (your monthly $250 additional investment) comes at the end of each period. (This step is necessary only if the **BEGIN** mode annunciator now appears in the display—you want to turn that annunciator off, to restore END mode.)
12 n	There are 12 periods (months) in this situation.
0.5 i	The 6% rate given is the nominal annualized rate; the actual defined interest rate is monthly: 6%/12, or 0.5% per month.
1000 CHS PV	The initial investment is **negative** because it's being **paid** by you.
250 CHS PMT	The PMT amount is also an **investment**—**paid** by you.
FV	Solve for Future Value to get the year-end balance: $**4,145.57** This is a **positive** value because you **receive** (withdraw) it.

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Notice the process for storing known values into the financial registers.

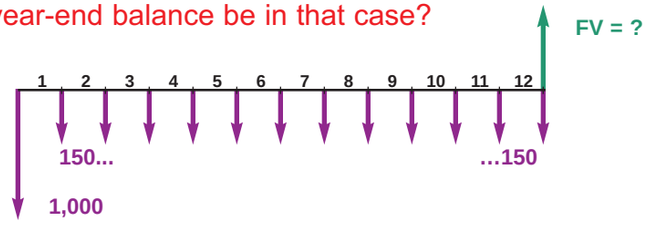
You keyed the values into the **X** register—where everything first enters the calculator. Then, you could have pressed [STO][n] or [STO][i] or [STO][PV] or [STO][PMT]—but you didn't! You skipped the [STO] key altogether. Why? Isn't this also the way to calculate the unknown value—just press the desired financial register key? (Yes, that's exactly how you computed the Future Value: just pressed [FV].)

The "shortcut storage" for the financial registers is simply a handy feature that HP provided because you use those particular registers so intensively. *But how does the calculator know that you intended to store—rather than calculate— those values?*

When you press any one of the five financial register keys, the calculator checks to see what keystroke you pressed *just prior to that*. If it was the [STO] key, of course that means you wish to store a value. But if the prior keystroke was *anything that placed a number into the **X** register*, this also is assumed to mean that you want to store that value into the financial register. If the prior keystroke was anything else, the HP 12C assumes you want to calculate that financial value. Smart machine.

Try another: Continuing from the previous example, "what if" your monthly investments were just $150 rather than? What would the year-end balance be in that case?

Solution: Don't erase everything! You need to change just one value—the PMT:



Keystrokes	Comments
150 [CHS] [PMT]	The new value simply replaces the old one. (And notice again the storage shortcut—courtesy of a smart HP 12C.)
[FV]	Now, as before, you solve for Future Value (and now the smart machine knows you wish to calculate) to get the end balance: $**2,912.01**. Playing "what if" is this easy!

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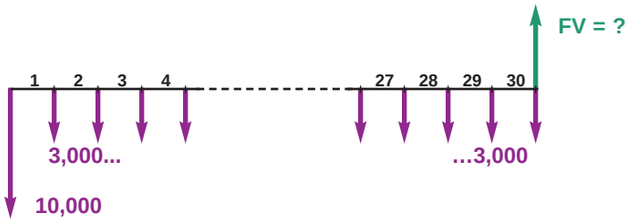
HP 12C Time Value of Money (TVM) Basics

Solving for Future Value (FV)

For more practice with financial (TVM) calculations on the HP 12C, look now at how you can examine an investment situation, retirement planning, and play "What if" with any of the five TVM values. Notice the directions of the cash flows (indicated by either **positive** or **negative** values. Note, too, that the payment mode for all solutions is END mode (no **BEGIN** annunciator), since each periodic investment occurs at the end of the corresponding period. (Press g END if needed.)



Original Problem: What's the resulting balance after 30 years if you open an investment account paying 10% annual interest with an initial investment of $10,000, and then make an additional investment of $3000 at the end of every year?

Solution: 30 n
10 i
10000 CHS PV
3000 CHS PMT
FV... Result: **667,976.09**

Problem: Same as the original problem, except what if you invest for 35 years instead of 30?

Solution: Continuing from before, change only what's different: 35 n
FV... Result: **1,094,097.47**

Problem: Same as the original problem, except what if the account pays 11% interest, not 10?

Solution: Continuing from before, change only what's different: 30 n 11 i
FV... Result: **825,985.60**

Problem: Same as the original problem, except what if your initial investment is only $5000?

Solution: Cont. from before, change only as needed: 10 i 5000 CHS PV
FV... Result: **580,729.08**

Problem: Same as the original problem, except what if your annual investment is only $2000?

Solution: Change only as necessary:
10000 CHS PV 2000 CHS PMT
FV... Result: **503,482.07**

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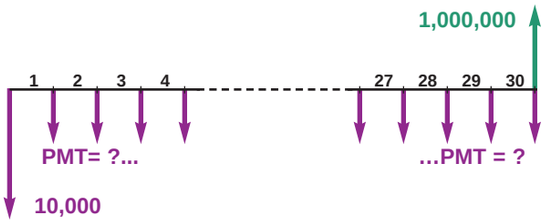
HP 12C Time Value of Money (TVM) Basics

Solving for Payment (PMT)

Here's the retirement situation again. Now you are trying to determine the periodic investment amount necessary for the retirement account, given various investment circumstances over the years. The HP 12C will compute that value as ***negative***; you're investing—**paying**—it each year. Note: END mode applies here. (Press g END if needed.)



Original Problem: What yearly investment is necessary over 30 years to retire with $1 million, if you open an account paying 10% annual interest, with an initial investment of 10,000, then make an additional investment at every year's end?

Solution: 30 n
10 i
10000 CHS PV
1000000 FV
PMT... Result: **-5,018.46**

Problem: Same as the original problem, except what if you invest for 35 years instead of 30?

Solution: Continuing from before, change only what's different: 35 n
PMT... Result: **-2,652.81**

Problem: Same as the original problem, except what if the account pays 11% interest, not 10?

Solution: Continuing from before, change only what's different: 30 n 11 i
PMT... Result: **-3,874.35**

Problem: Same as the original problem, except what if your initial investment is only $5000?

Solution: Cont. from previous, change only as needed: 10 i 5000 CHS PV
PMT... Result: **-5,548.85**

Problem: Same as the original problem, except what if your retirement goal is only $750,000?

Solution: Change only as needed:
10000 CHS PV 750000 FV
PMT... Result: **-3,498.64**

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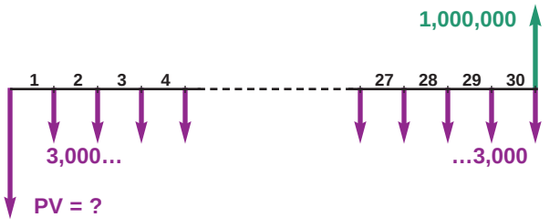
HP 12C Time Value of Money (TVM) Basics

Solving for Present Value (PV)

Here's a picture of the retirement situation again. Now you're trying to determine the initial investment amount necessary for the retirement account, given various circumstances over the years. (The calculator will compute that value as ***negative***; you're investing—**paying**—it to open the account.) END mode applies. (Press g END if needed.)



Original Problem: What initial investment is necessary to retire with $1,000,000, if, after you open an account paying 10% annual interest with that initial investment, you then make an an additional investment of $3000 every year?

Solution: 30 n
10 i
3000 CHS PMT
1000000 FV
PV… Result: **-29,027.81**

Problem: Same as the original problem, except what if you invest for 35 years instead of 30?

Solution: Continuing from before, change only what's different: 35 n
PV… Result: **-6,651.63**

Problem: Same as the original problem, except what if the account pays 11% interest, not 10?

Solution: Continuing from before, change only what's different: 30 n 11 i
PV… Result: **-17,601.44**

Problem: Same as the original problem, except what if your annual investment is only $2000?

Solution: Cont. from before, change just if needed: 10 i 2000 CHS PMT
PV… Result: **-38,454.72**

Problem: Same as the original problem, except what if your retirement goal is $750,000?

Solution: Change only as needed:
3000 CHS PMT 750000 FV
PV… Result: **-14,700.67**

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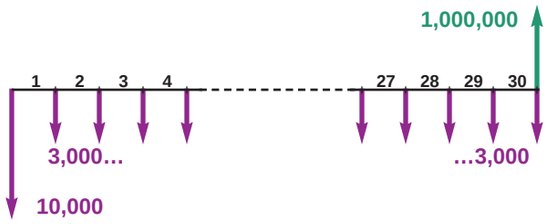
HP 12C Time Value of Money (TVM) Basics

Solving for Interest Rate (i)

Here's a picture of the retirement situation again. Now you're trying to determine the annual interest rate necessary for the retirement account, given various circumstances over the years. (The calculator will take some time to compute the rate.) Note: END mode applies. (Press g END if needed.)



Original Problem: What annual interest rate is necessary in order to retire with $1 million if, after you open an account (which earns that rate) with an initial investment of $10,000, you make an additional annual investment of $3000?

Solution: 30 n
10000 CHS PV
3000 CHS PMT
1000000 FV
i... Result: **11.89**

Problem: Same as the original problem, except what if you invest for 35 years instead of 30?

Solution: Continuing from before, change only what's different: 35 n
i... Result: **9.64**

Problem: Same as the original problem, except what if your initial investment is only $5000?

Solution: Cont. from before, change only as needed: 30 n 5000 CHS PV
i... Result: **12.65**

Problem: Same as the original problem, except what if your annual investment is only $2000?

Solution: Change values only as needed:
10000 CHS PV 2000 CHS PMT
i... Result: **13.10**

Problem: Same as the original problem, except what if your retirement goal is $750,000?

Solution: Change only as needed:
3000 CHS PMT 750000 FV
i... Result: **10.55**

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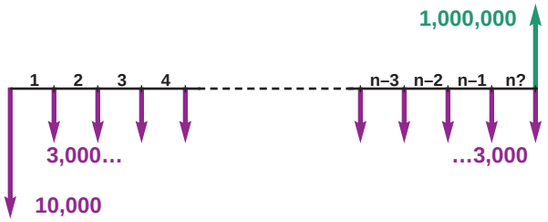
HP 12C Time Value of Money (TVM) Basics

Solving for Number of Periods (n)

Here's a picture of the retirement situation again. Now you're trying to determine the number of years necessary for you to pay into the retirement account, given various circumstances over the years. (Note: The calculator will round the result *up*—see the discussion following the solutions.)
END mode applies here. (Press g END if needed.)



Original Problem: How many years of investing are necessary in order to retire with $1 million, if, after you open an account earning 10% annual interest, with an initial investment of $10,000, you make an additional annual investment of $3000?

Solution: 10 i
10000 CHS PV
3000 CHS PMT
1000000 FV
n... Result: **35.00**

Problem: Same as the original problem, except what if the account earns 11% annual interest?

Solution: Continuing from before, change only what's different: 11 i
n... Result: **32.00**

Problem: Same as the original problem, except what if your initial investment is only $5000?

Solution: Cont. from previous, change only as needed: 10 i 5000 CHS PV
n... Result: **36.00**

Problem: Same as the original problem, except what if your annual investment is only $2000?

Solution: Change values only as needed:
10000 CHS PV 2000 CHS PMT
n... Result: **37.00**

Problem: Same as the original problem, except what if your retirement goal is $750,000?

Solution: Change only as needed:
3000 CHS PMT 750000 FV
n... Result: **32.00**

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HP 12C Time Value of Money (TVM) Basics

Solving for n (number of periods) on the HP 12C gives you only an approximate value; the number of periods is rounded up to the next whole period. Thus, when you get a result, you know only that the number of periods required for the situation is somewhere between n and n-1.

Why does the HP 12C do this? It's because the math that allows you to solve TVM problems really only applies when n is a whole number. Yes, a lot of actuarial calculations use partial periods—and other calculators produce them—but when PMT is not zero, exactly what does that fractional portion of the n value really mean? Does it mean that a partial PMT will be paid? If so, when? Before or after a partial period? And when does that period occur—before or after all the normal whole periods? Mathematically, none of these interpretations is correct; the n value you get simply by using the standard TVM formula *has no meaning when it is not a whole number*. So the HP 12C gives you a rounded result.

If you want to know a non-integer exact result, you'll have to key in a guess for that value (somewhere between the rounded value you calculated and 1 less than that) and re-calculate Future Value (or one of the other known TVM variables in the problem—FV is probably the easiest). But note that the HP 12C will make certain assumptions about when and how to take that partial period into account:

To remove the ambiguity inherent in a non-integer n value, HP has modified the standard TVM formula so that whenever you *key in* a non-integer (non-whole-number) value for n, the HP 12C will interpret this very specifically: The partial period comes at the *beginning* of the timeline (i.e. after PV but before the first whole period), and there is no "partial PMT" associated with this partial period. (You are also given a choice as to how you wish the interest to accrue on PV during that partial period. The default choice—because it is more common—is straight-line, or simple interest. The other choice—a mode in effect when the little **c** annunciator is showing in the display—is Continuously compounding interest.)

So, for example, if you really want to know "exactly" how long it will take to accrue a million dollars in your retirement account earning 10%, with an initial investment of $10,000 and an additional annual investment of $3000, you would have to proceed as follows:

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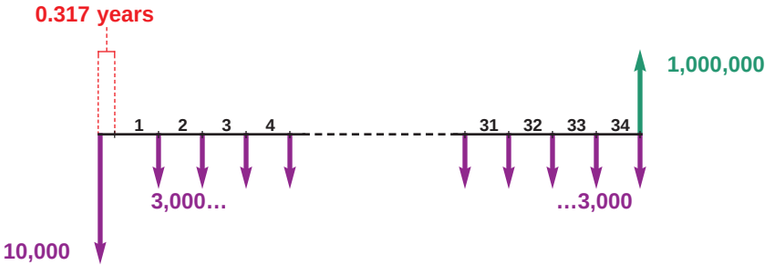
First, get the original solution: 10 [i] 10000 [CHS] [PV] 3000 [CHS] [PMT] 1000000 [FV] [n]…
Result: **35.00**

But that's *rounded*. You know only that the actual value is somewhere between 34 and 35 years. So key in a guess—say, 34.5 years—and solve for the Future Value: 34.5 [n] [FV]…
Result: **1,004,680.63**

That's too high for the desired Future Value ($1 million exactly), so it's slightly too high a value for n (i.e. too long a time). So try a lesser guess: 34.3 [n] [FV]…
Result: **99,571.10**

Well, that's a little too low—but not by much. So now you know the actual n value is somewhere between 34.3 and 34.5 (and closer to 34.3).

And of course, you could continue in this fashion to get as exact an n value as you want (it's about 34.317 years). The point is, *what exactly does that n value mean?* Under the assumptions the HP 12C makes, it means this—and only this:



In other words, to accrue exactly a million dollars in the 10% account, starting with an initial investment of $10,000, you'd make that initial investment, then wait for .317 years (a little less than four months) before your first full investment year would actually begin—and then you would wait a full year more from that point until your first annual investment (PMT) of $3000. Thus there are only 34 such annual investments (not 35—and certainly not 34.317). The partial period is placed before the first PMT ever happens—so that the extra time accrues interest on only the PV amount—your initial investment ($10,000).